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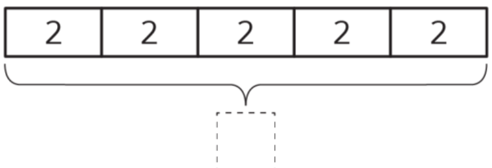
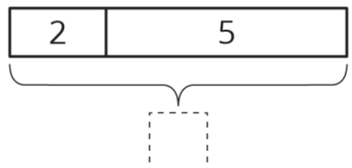
Unit 6, Lesson 1: Tape Diagrams and Equations

Let's see how tape diagrams and equations can show relationships between amounts.

1.1: Which Diagram is Which?

Here are two diagrams. One represents $2 + 5 = 7$. The other represents $5 \cdot 2 = 10$. Which is which? Label the length of each diagram.



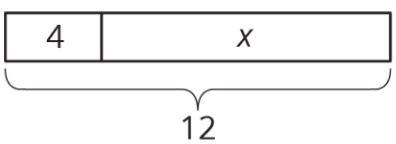


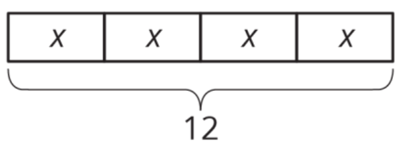
Draw a diagram that represents each equation.

1. $4 + 3 = 7$
2. $4 \cdot 3 = 12$

1.2: Match Equations and Tape Diagrams

Here are two tape diagrams. Match each equation to one of the tape diagrams.





1. $4 + x = 12$
2. $12 \div 4 = x$
3. $4 \cdot x = 12$
4. $12 = 4 + x$
5. $12 - x = 4$
6. $12 = 4 \cdot x$
7. $12 - 4 = x$
8. $x = 12 - 4$
9. $x + x + x + x = 12$

1.3: Draw Diagrams for Equations

For each equation, draw a diagram and find the value of the unknown that makes the equation true.

1. $18 = 3 + x$
2. $18 = 3 \cdot y$

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Are you ready for more?

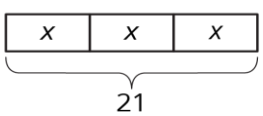
You are walking down a road, seeking treasure. The road branches off into three paths. A guard stands in each path. You know that only one of the guards is telling the truth, and the other two are lying. Here is what they say:

- Guard 1: The treasure lies down this path.
- Guard 2: No treasure lies down this path; seek elsewhere.
- Guard 3: The first guard is lying.

Which path leads to the treasure?

Lesson 1 Summary

Tape diagrams can help us understand relationships between quantities and how operations describe those relationships.

A


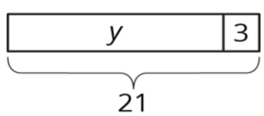
B


Diagram A has 3 parts that add to 21. Each part is labeled with the same letter, so we know the three parts are equal. Here are some equations that all represent diagram A:

$x + x + x = 21$

$3 \cdot x = 21$

$x = 21 \div 3$

$x = \frac{1}{3} \cdot 21$

Notice that the number 3 is not seen in the diagram; the 3 comes from counting 3 boxes representing 3 equal parts in 21.

We can use the diagram or any of the equations to reason that the value of x is 7.

Diagram B has 2 parts that add to 21. Here are some equations that all represent diagram B:

$y + 3 = 21$

$y = 21 - 3$

$3 = 21 - y$

We can use the diagram or any of the equations to reason that the value of y is 18.

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Unit 6, Lesson 2: Truth and Equations

Let's use equations to represent stories and see what it means to solve equations.

2.1: Three Letters

1. The equation $a + b = c$ could be true or false.
    a. If a is 3, b is 4, and c is 5, is the equation true or false?
    b. Find new values of a, b, and c that make the equation true.
    c. Find new values of a, b, and c that make the equation false.
2. The equation $x \cdot y = z$ could be true or false.
    a. If x is 3, y is 4, and z is 12, is the equation true or false?
    b. Find new values of x, y, and z that make the equation true.
    c. Find new values of x, y, and z that make the equation false.

2.2: Storytime

Here are three situations and six equations. Which equation best represents each situation? If you get stuck, draw a diagram.

1. After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. She ran x miles before Friday.
2. Andre's school has 20 clubs, which is five times as many as his cousin's school. His cousin's school has x clubs.
3. Jada volunteers at the animal shelter. She divided 5 cups of cat food equally to feed 20 cats. Each cat received x cups of food.

$x + 5 = 20$	$x = 20 + 5$	$5x = 20$
$x + 20 = 5$	$5 \cdot 20 = x$	$20x = 5$

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2.3: Using Structure to Find Solutions

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, draw a diagram. Be prepared to explain why your solution is correct.

1. $1000 - a = 400$
2. $12.6 = b + 4.1$
3. $8c = 8$
4. $\frac{2}{3} \cdot d = \frac{10}{9}$
5. $10e = 1$
6. $10 = 0.5f$
7. $0.99 = 1 - g$
8. $h + \frac{3}{7} = 1$

List:

$\frac{1}{8}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{7}{3}$	0.01	0.1	0.5
1	2	8.5	9.5	16.7	20	400	600	1400

Are you ready for more?

One solution to the equation $a + b + c = 10$ is $a = 2$, $b = 5$, $c = 3$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

Lesson 2 Summary

An equation can be true or false. An example of a true equation is $7 + 1 = 4 \cdot 2$. An example of a false equation is $7 + 1 = 9$.

An equation can have a letter in it, for example, $u + 1 = 8$. This equation is false if u is 3, because $3 + 1$ does not equal 8. This equation is true if u is 7, because $7 + 1 = 8$.

A letter in an equation is called a **variable.** In $u + 1 = 8$, the variable is u. A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. In $u + 1 = 8$, the solution is 7.

When a number is written next to a variable, the number and the variable are being multiplied. For example, $7x = 21$ means the same thing as $7 \cdot x = 21$. A number written next to a variable is called a **coefficient**. If no coefficient is written, the coefficient is 1. For example, in the equation $p + 3 = 5$, the coefficient of p is 1.

Lesson 2 Glossary Terms

solution to an equation
variable
coefficient

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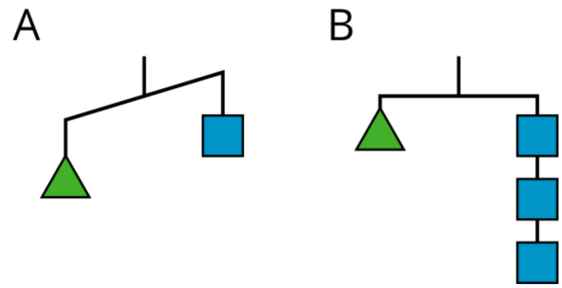
Unit 6, Lesson 3: Staying in Balance

Let's use balanced hangers to help us solve equations.

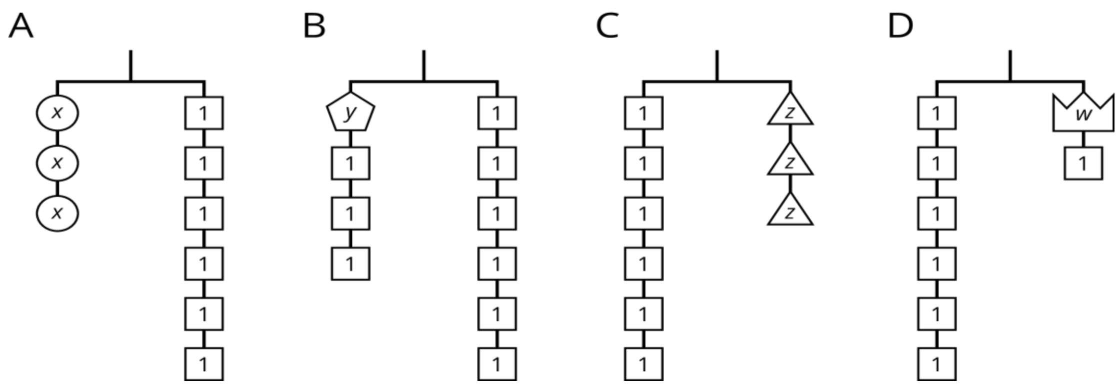


3.1: Hanging Around

1. For diagram A, find:
    1. One thing that *must* be true
    2. One thing that *could* be true or false
    3. One thing that *cannot possibly* be true

2. For diagram B, find:
    1. One thing that *must* be true
    2. One thing that *could* be true or false
    3. One thing that *cannot possibly* be true

3.2: Match Equations and Hangers



1. Match each hanger to an equation. Complete the equation by writing x, y, z, or w in the empty box.

   $\square + 3 = 6$ $3 \cdot \square = 6$ $6 = \square + 1$ $6 = 3 \cdot \square$

2. Find a solution to each equation. Use the hangers to explain what each solution means.

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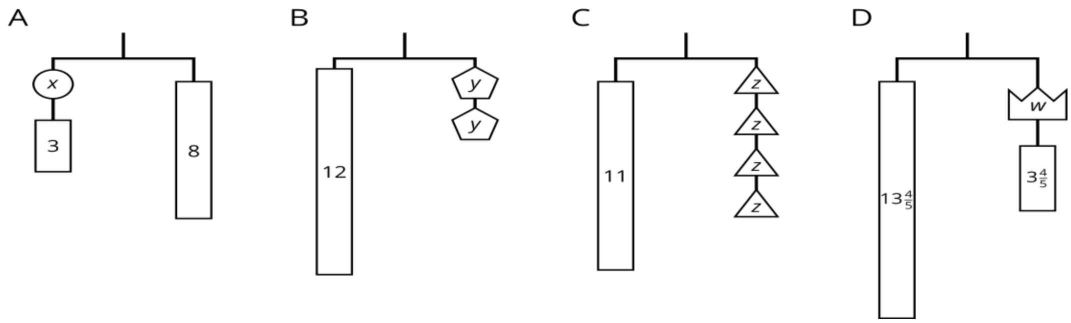
3.3: Connecting Diagrams to Equations and Solutions

Here are some balanced hangers. Each piece is labeled with its weight.



For each diagram:

1. Write an equation.
2. Explain how to reason with the diagram to find the weight of a piece with a letter.
3. Explain how to reason with the equation to find the weight of a piece with a letter.

Diagram A:

Diagram B:

Diagram C:

Diagram D:

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Lesson 3 Summary

A hanger stays balanced when the weights on both sides are equal. We can change the weights and the hanger will stay balanced as long as both sides are changed in the same way. For example, adding 2 pounds to each side of a balanced hanger will keep it balanced. Removing half of the weight from each side will also keep it balanced.

An equation can be compared to a balanced hanger. We can change the equation, but for a true equation to remain true, the same thing must be done to both sides of the equal sign. If we add or subtract the same number on each side, or multiply or divide each side by the same number, the new equation will still be true.

This way of thinking can help us find solutions to equations. Instead of checking different values, we can think about subtracting the same amount from each side or dividing each side by the same number.

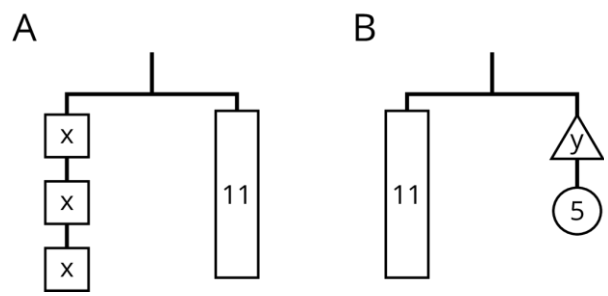

Diagram A can be represented by the equation $3x = 11$.

If we break the 11 into 3 equal parts, each part will have the same weight as a block with an x.

Splitting each side of the hanger into 3 equal parts is the same as dividing each side of the equation by 3.

- $3x$ divided by 3 is x.
- 11 divided by 3 is $\frac{11}{3}$.
- If $3x = 11$ is true, then $x = \frac{11}{3}$ is true.
- The solution to $3x = 11$ is $\frac{11}{3}$.

Diagram B can be represented with the equation $11 = y + 5$.

If we remove a weight of 5 from each side of the hanger, it will stay in balance.

Removing 5 from each side of the hanger is the same as subtracting 5 from each side of the equation.

- $11 - 5$ is 6.
- $y + 5 - 5$ is y.
- If $11 = y + 5$ is true, then $6 = y$ is true.
- The solution to $11 = y + 5$ is 6.

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Unit 6, Lesson 4: Practice Solving Equations and Representing Situations with Equations

Let's solve equations by doing the same to each side.

4.1: Number Talk: Subtracting From Five

Find the value of each expression mentally.

$5 - 2$ $\quad$ $5 - 2.1$ $\quad$ $5 - 2.17$ $\quad$ $5 - 2\frac{7}{8}$

4.2: Row Game: Solving Equations Practice

Solve the equations in one column. Your partner will work on the other column.

Check in with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error and correct it.

<u>**WORK SPACE**</u>

column A	column B
$18 = 2x$	$36 = 4x$
$17 = x + 9$	$13 = x + 5$
$8x = 56$	$3x = 21$
$21 = \frac{1}{4}x$	$28 = \frac{1}{3}x$
$6x = 45$	$8x = 60$
$x + 4\frac{5}{6} = 9$	$x + 3\frac{5}{6} = 8$
$\frac{5}{7}x = 55$	$\frac{3}{7}x = 33$
$\frac{1}{5} = 6x$	$\frac{1}{3} = 10x$
$2.17 + x = 5$	$6.17 + x = 9$
$\frac{20}{3} = \frac{10}{9}x$	$\frac{14}{5} = \frac{7}{15}x$
$14.88 + x = 17.05$	$3.91 + x = 6.08$
$3\frac{3}{4}x = 1\frac{1}{4}$	$\frac{7}{5}x = \frac{7}{15}$

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4.3: Choosing Equations to Match Situations

- Circle **all** of the equations that describe each situation. If you get stuck, draw a diagram.
- Find the solution for each situation.

1. Clare has 8 fewer books than Mai. If Mai has 26 books, how many books does Clare have? ○ $26 - x = 8$ ○ $x = 26 + 8$ ○ $x + 8 = 26$ ○ $26 - 8 = x$ $x =$ ________	2. A coach formed teams of 8 from all the players in a soccer league. There are 14 teams. How many players are in the league? ○ $y = 14 \div 8$ ○ $\frac{y}{8} = 14$ ○ $\frac{1}{8}y = 14$ ○ $y = 14 \cdot 8$ $y =$ ________
3. Kiran scored 223 more points in a computer game than Tyler. If Kiran scored 409 points, how many points did Tyler score? ○ $223 = 409 - z$ ○ $409 - 223 = z$ ○ $409 + 223 = z$ ○ $409 = 223 + z$ $z =$ ________	4. Mai ran 27 miles last week, which was three times as far as Jada ran. How far did Jada run? ○ $3w = 27$ ○ $w = \frac{1}{3} \cdot 27$ ○ $w = 27 \div 3$ ○ $w = 3 \cdot 27$ $w =$ ________

Are you ready for more?

Mai's mother was 28 when Mai was born. Mai is now 12 years old. In how many years will Mai's mother be twice Mai's age? How old will they be then?

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Lesson 4 Summary

Writing and solving equations can help us answer questions about situations.

Suppose a scientist has 13.68 liters of acid and needs 16.05 liters for an experiment. How many more liters of acid does she need for the experiment?

- We can represent this situation with the equation:

$$13.68 + x = 16.05$$

- When working with hangers, we saw that the solution can be found by subtracting 13.68 from each side. This gives us some new equations that also represent the situation:

$$x = 16.05 - 13.68$$
$$x = 2.37$$

- Finding a solution in this way leads to a variable on one side of the equal sign and a number on the other. We can easily read the solution—in this case, 2.37—from an equation with a letter on one side and a number on the other. We often write solutions in this way.

Let's say a food pantry takes a 54-pound bag of rice and splits it into portions that each weigh $\frac{3}{4}$ of a pound. How many portions can they make from this bag?

- We can represent this situation with the equation:

$$\frac{3}{4}x = 54$$

- We can find the value of x by dividing each side by $\frac{3}{4}$. This gives us some new equations that represent the same situation:

$$x = 54 \div \frac{3}{4}$$
$$x = 72$$

- The solution is 72 portions.

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Unit 6, Lesson 5: A New Way to Interpret a over b

Let's investigate what a fraction means when the numerator and denominator are not whole numbers.

5.1: Recalling Ways of Solving

Solve each equation. Be prepared to explain your reasoning.

1. $0.07 = 10m$
2. $10.1 = t + 7.2$

5.2: Interpreting $\frac{a}{b}$

Solve each equation.

1. $35 = 7x$
2. $35 = 11x$
3. $7x = 7.7$
4. $0.3x = 2.1$
5. $\frac{2}{5} = \frac{1}{2}x$

Are you ready for more?

Solve the equation. Try to find some shortcuts.

$$\frac{1}{6} \cdot \frac{3}{20} \cdot \frac{5}{42} \cdot \frac{7}{72} \cdot x = \frac{1}{384}$$

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5.3: Storytime Again

Take turns with your partner telling a story that might be represented by each equation. Then, for each equation, choose one story, state what quantity x describes, and solve the equation. If you get stuck, draw a diagram.

1. $0.7 + x = 12$
2. $\frac{1}{4}x = \frac{3}{2}$

Lesson 5 Summary

In the past, you learned that a fraction such as $\frac{4}{5}$ can be thought of in a few ways.

- $\frac{4}{5}$ is a number you can locate on the number line by dividing the section between 0 and 1 into 5 equal parts and then counting 4 of those parts to the right of 0.
- $\frac{4}{5}$ is the share that each person would have if 4 wholes were shared equally among 5 people. This means that $\frac{4}{5}$ is the result of *dividing* 4 by 5.

We can extend this meaning of *a fraction as a division* to fractions whose numerators and denominators are not whole numbers. For example, we can represent 4.5 pounds of rice divided into portions that each weigh 1.5 pounds as: $\frac{4.5}{1.5} = 4.5 \div 1.5 = 3$.

Fractions that involve non-whole numbers can also be used when we solve equations.

Suppose a road under construction is $\frac{3}{8}$ finished and the length of the completed part is $\frac{4}{3}$ miles. How long will the road be when completed?

We can write the equation $\frac{3}{8}x = \frac{4}{3}$ to represent the situation and solve the equation.

The completed road will be $3\frac{5}{9}$ or about 3.6 miles long.

$$\begin{aligned} \frac{3}{8}x &= \frac{4}{3} \\ x &= \frac{\frac{4}{3}}{\frac{3}{8}} \\ x &= \frac{4}{3} \cdot \frac{8}{3} \\ x &= \frac{32}{9} = 3\frac{5}{9} \end{aligned}$$

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Unit 6, Lesson 6: Write Expressions Where Letters Stand for Numbers

Let's use expressions with variables to describe situations.

6.1: Algebra Talk: When x is 6

If x is 6, what is:

$x+4$ $\quad$ $7-x$ $\quad$ x^2 $\quad$ $\frac{1}{3}x$

6.2: Lemonade Sales and Heights

1. Lin set up a lemonade stand. She sells the lemonade for $0.50 per cup.

   a. Complete the table to show how much money she would collect if she sold each number of cups.

lemonade sold (number of cups)	12	183	c
money collected (dollars)			

   b. How many cups did she sell if she collected $127.50? Be prepared to explain your reasoning.

2. Elena is 59 inches tall. Some other people are taller than Elena.

   a. Complete the table to show the height of each person.

person	Andre	Lin	Noah
how much taller than Elena (inches)	4	$6\frac{1}{2}$	d
person's height (inches)			

   b. If Noah is $64\frac{3}{4}$ inches tall, how much taller is he than Elena?

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6.3: Building Expressions

1. Clare is 5 years older than her cousin.

   a. How old would Clare be if her cousin is:

   10 years old? 2 years old? x years old?

   b. Clare is 12 years old. How old is Clare's cousin?

2. Diego has 3 times as many comic books as Han.

   a. How many comic books does Diego have if Han has:

   6 comic books? n books?

   b. Diego has 27 comic books. How many comic books does Han have?

3. Two fifths of the vegetables in Priya's garden are tomatoes.

   a. How many tomatoes are there if Priya's garden has:

   20 vegetables? x vegetables?

   b. Priya's garden has 6 tomatoes. How many total vegetables are there?

4. A school paid \$31.25 for each calculator.

   a. If the school bought x calculators, how much did they pay?

   b. The school spent \$500 on calculators. How many did the school buy?

Are you ready for more?

Kiran, Mai, Jada, and Tyler went to their school carnival. They all won chips that they could exchange for prizes. Kiran won $\frac{2}{3}$ as many chips as Jada. Mai won 4 times as many chips as Kiran. Tyler won half as many chips as Mai.

1. Write an expression for the number of chips Tyler won. You should only use one variable: J, which stands for the number of chips Jada won.

2. If Jada won 42 chips, how many chips did Tyler, Kiran, and Mai each win?

Lesson 6 Summary

Suppose you share a birthday with a neighbor, but she is 3 years older than you. When you were 1, she was 4. When you were 9, she was 12. When you are 42, she will be 45.

If we let a represent your age at any time, your neighbor’s age can be expressed $a + 3$.

your age	1	9	42	a
neighbor's age	4	12	45	$a+3$

We often use a letter such as x or a as a placeholder for a number in expressions. These are called *variables* (just like the letters we used in equations, previously). Variables make it possible to write expressions that represent a calculation even when we don't know all the numbers in the calculation.

How old will you be when your neighbor is 32? Since your neighbor's age is calculated with the expression $a + 3$, we can write the equation $a + 3 = 32$. When your neighbor is 32 you will be 29, because $a + 3 = 32$ is true when a is 29.

Unit 6, Lesson 7: Revisit Percentages

Let's use equations to find percentages.

7.1: Number Talk: Percentages

Solve each problem mentally.

1. Bottle A contains 4 ounces of water, which is 25% of the amount of water in Bottle B. How much water is there in Bottle B?

2. Bottle C contains 150% of the water in Bottle B. How much water is there in Bottle C?

3. Bottle D contains 12 ounces of water. What percentage of the amount of water in Bottle B is this?

7.2: Representing a Percentage Problem with an Equation

1. Answer each question and show your reasoning.

   a. Is 60% of 400 equal to 87?

   b. Is 60% of 200 equal to 87?

   c. Is 60% of 120 equal to 87?

2. 60% of x is equal to 87. Write an equation that expresses the relationship between 60%, x, and 87. Solve your equation.

3. Write an equation to help you find the value of each variable. Solve the equation.

    a. 60% of c is 43.2.

    b. 38% of e is 190.

7.3: Puppies Grow Up, Revisited

1. Puppy A weighs 8 pounds, which is about 25% of its adult weight. What will be the adult weight of Puppy A?

2. Puppy B weighs 8 pounds, which is about 75% of its adult weight. What will be the adult weight of Puppy B?

3. If you haven't already, write an equation for each situation. Then, show how you could find the adult weight of each puppy by solving the equation.

Are you ready for more?

Diego wants to paint his room purple. He bought one gallon of purple paint that is 30% red paint and 70% blue paint. Diego wants to add more blue to the mix so that the paint mixture is 20% red, 80% blue.

1. How much blue paint should Diego add? Test the following possibilities: 0.2 gallons, 0.3 gallons, 0.4 gallons, 0.5 gallons.

2. Write an equation in which x represents the amount of paint Diego should add.

3. Check that the amount of paint Diego should add is a solution to your equation.

Lesson 7 Summary

If we know that 455 students are in school today and that number represents 70% attendance, we can write an equation to figure out how many students go to the school.

The number of students in school today is known in two different ways: as 70% of the students in the school, and also as 455. If s represents the total number of students who go to the school, then 70% of s, or $\frac{70}{100}s$, represents the number of students that are in school today, which is 455.

We can write and solve the equation:

$$\begin{aligned} \frac{70}{100}s &= 455 \\ s &= 455 \div \frac{70}{100} \\ s &= 455 \cdot \frac{100}{70} \\ s &= 650 \end{aligned}$$

There are 650 students in the school.

In general, equations can help us solve problems in which one amount is a percentage of another amount.

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Unit 6, Lesson 8: Equal and Equivalent

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

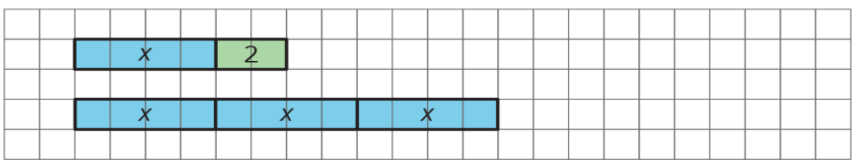
8.1: Algebra Talk: Solving Equations by Seeing Structure

Find a solution to each equation mentally.

$3 + x = 8$ $\qquad 10 = 12 - x$ $\qquad x^2 = 49$ $\qquad \frac{1}{3}x = 6$

8.2: Using Diagrams to Show That Expressions are Equivalent

Here is a diagram of $x + 2$ and $3x$ when x is 4. Notice that the two diagrams are lined up on their left sides.



In each of your drawings below, line up the diagrams on one side.

1. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 3.

2. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 2.

3. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 1.

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4. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 0.

5. When are $x + 2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.

6. Draw a diagram of $x + 3$, and a separate diagram of $3 + x$.

7. When are $x + 3$ and $3 + x$ equal? When are they not equal? Use your diagrams to explain.

8.3: Identifying Equivalent Expressions

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, try reasoning with diagrams.

$a + 3$	$a \div \frac{1}{3}$	$\frac{1}{3}a$	$\frac{a}{3}$	a
$a + a + a$	$a \cdot 3$	$3a$	$1a$	$3 + a$

Are you ready for more?

Below are four questions about equivalent expressions. For each one:

- Decide whether you think the expressions are equivalent.
- Test your guess by choosing numbers for x (and y, if needed).

1. Are $\frac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?

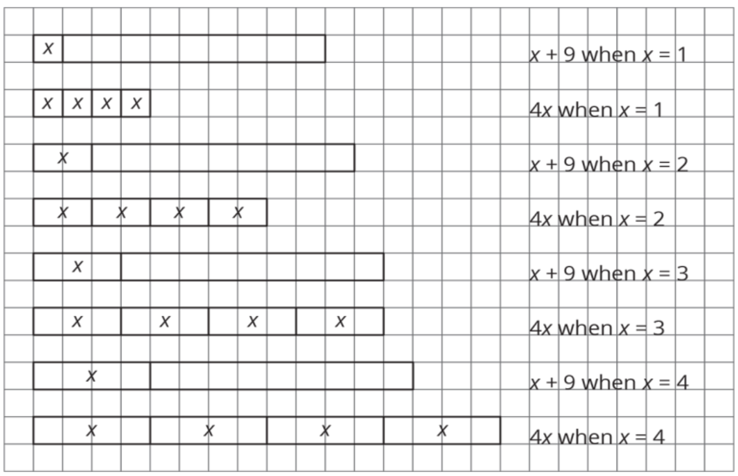
2. Are $\frac{x+x+x+x}{x}$ and $x+x+x$ equivalent expressions?

3. Are $2(x+y)$ and $2x+2y$ equivalent expressions?

4. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

Lesson 8 Summary

We can use diagrams showing lengths of rectangles to see when expressions are equal. For example, the expressions $x+9$ and $4x$ are equal when x is 3, but are not equal for other values of x.



Sometimes two expressions are equal for only one particular value of their variable. Other times, they seem to be equal no matter what the value of the variable.

Expressions that are always equal for the same value of their variable are called **equivalent expressions.** However, it would be impossible to test every possible value of the variable. How can we know for sure that expressions are equivalent?

We use the meaning of operations and properties of operations to know that expressions are equivalent. Here are some examples:

- $x+3$ is equivalent to $3+x$ because of the commutative property of addition.
- $4 \cdot y$ is equivalent to $y \cdot 4$ because of the commutative property of multiplication.
- $a+a+a+a+a$ is equivalent to $5 \cdot a$ because adding 5 copies of something is the same as multiplying it by 5.
- $b \div 3$ is equivalent to $b \cdot \frac{1}{3}$ because dividing by a number is the same as multiplying by its reciprocal.

In the coming lessons, we will see how another property, the distributive property, can show that expressions are equivalent.

NAME DATE PERIOD

CPM Lesson 7.3.1: Why does it work?

Inverse Operations

7-79.

Steps	Trial 1	Trial 2	Trial 3
1. Pick a number.			
2. Add 5.			
3. Double it.			
4. Subtract 4.			
5. Divide by 2.			
6. Subtract the original number.			

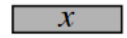
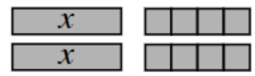
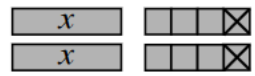
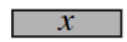
7-81.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number.				
2. Add 2.				
3. Multiply by 3.				
4. Subtract 3.				
5. Divide by 3.				
6. Subtract the original number.				

NAME DATE PERIOD

CPM Lesson 7.3.1: Why does it work?

7-82.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number.				
2.				
3.				
4.				
5.				
6.				

7-84.

Steps	Trial 1	Trial 2	Trial 3	Algebra Tile Picture
1. Pick a number.				
2. Double it.				
3. Add 4.				
4. Multiply by 2.				
5. Divide by 4.				
6. Subtract the original number.				

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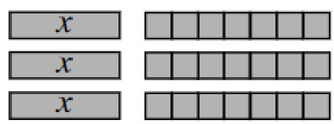
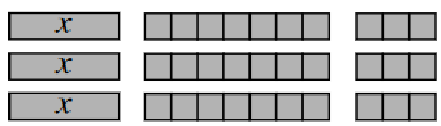
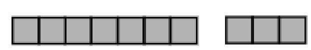
CPM Lesson 7.3.2: How can I write it?

Distributive Property

7-91.

Steps	Trial 1	Trial 2	Algebra Tile Picture	Algebraic Expression
1. Pick a number.				
2. Add 7.				
3. Triple the result.				
4. Add 9.				
5. Divide by 3.				
6. Subtract the original number.				

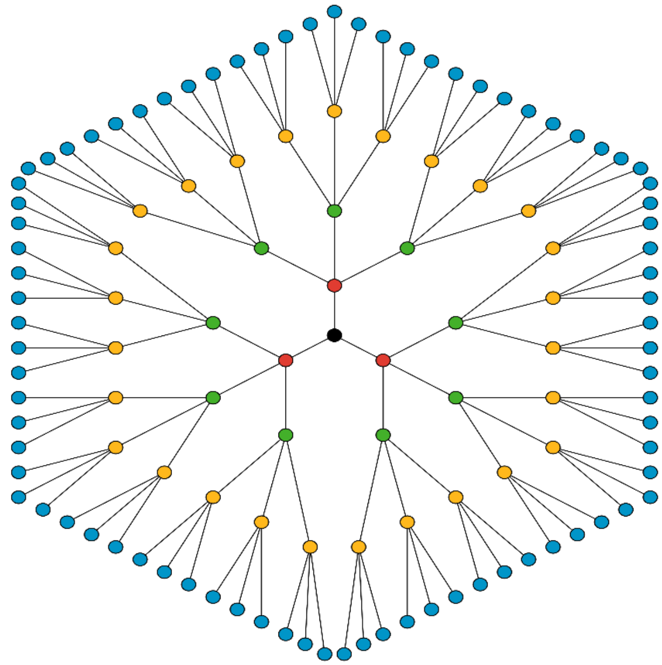
NAME DATE PERIOD

Unit 6, Lesson 12: Meaning of Exponents

Let's see how exponents show repeated multiplication.

12.1: Notice and Wonder: Dots and Lines

What do you notice? What do you wonder?



12.2: The Genie's Offer

You find a brass bottle that looks really old. When you rub some dirt off of the bottle, a genie appears! The genie offers you a reward. You must choose one:

- $50,000, or
- A magical $1 coin. The coin will turn into two coins on the first day. The two coins will turn into four coins on the second day. The four coins will double to 8 coins on the third day. The genie explains the doubling will continue for 28 days.

1. The number of coins on the third day will be $2 \cdot 2 \cdot 2$. Write an equivalent expression using exponents.

2. What do 2^5 and 2^6 represent in this situation? Evaluate 2^5 and 2^6 without a calculator. Pause for discussion.

3. How many days would it take for the number of magical coins to exceed $50,000?

4. Will the value of the magical coins exceed a million dollars within the 28 days? Explain or show your reasoning.

Are you ready for more?

A scientist is growing a colony of bacteria in a petri dish. She knows that the bacteria are growing and that the number of bacteria doubles every hour.

When she leaves the lab at 5 p.m., there are 100 bacteria in the dish. When she comes back the next morning at 9 a.m., the dish is completely full of bacteria. At what time was the dish half full?

12.3: Make 81

1. Here are some expressions. All but one of them equals 16. Find the one that is *not* equal to 16 and explain how you know.

   $2^3 \cdot 2$ $\qquad$ 4^2 $\qquad$ $\frac{2^5}{2}$ $\qquad$ 8^2

2. Write three expressions containing exponents so that each expression equals 81.

Lesson 12 Summary

When we write an expression like 2^n, we call n the exponent.

If n is a positive whole number, it tells how many factors of 2 we should multiply to find the value of the expression. For example, $2^1 = 2$, and $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$.

There are different ways to say 2^5. We can say “two raised to the power of five” or “two to the fifth power” or just “two to the fifth.”

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Unit 6, Lesson 13: Expressions with Exponents

Let's use the meaning of exponents to decide if equations are true.

13.1: Which One Doesn't Belong: Twos

Which one doesn't belong?

$2 \cdot 2 \cdot 2 \cdot 2$	2^4
16	$4 \cdot 2$

13.2: Is the Equation True?

Decide whether each equation is true or false, and explain how you know.

1. $2^4 = 2 \cdot 4$
2. $3 + 3 + 3 + 3 + 3 = 3^5$
3. $5^3 = 5 \cdot 5 \cdot 5$
4. $2^3 = 3^2$
5. $16^1 = 8^2$
6. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$
7. $\left(\frac{1}{2}\right)^4 = \frac{1}{8}$
8. $8^2 = 4^3$

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13.3: What's Your Reason?

In each list, find expressions that are equivalent to each other and explain to your partner why they are equivalent. Your partner listens to your explanation. If you disagree, explain your reasoning until you agree. Switch roles for each list.

(There may be more than two equivalent expressions in each list.)

1.
   a. $5 \cdot 5$
   b. 2^5
   c. 5^2
   d. $2 \cdot 5$

2.
   a. 4^3
   b. 3^4
   c. $4 \cdot 4 \cdot 4$
   d. $4 + 4 + 4$

3.
   a. $6 + 6 + 6$
   b. 6^3
   c. 3^6
   d. $3 \cdot 6$

4.
   a. 11^5
   b. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
   c. $11 \cdot 5$
   d. 5^{11}

5.
   a. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$
   b. $\left(\frac{1}{5}\right)^3$
   c. $\frac{1}{15}$
   d. $\frac{1}{125}$

6.
   a. $\left(\frac{5}{3}\right)^2$
   b. $\left(\frac{3}{5}\right)^2$
   c. $\frac{10}{6}$
   d. $\frac{25}{9}$

Are you ready for more?

What is the last digit of $3^{1,000}$? Show or explain your reasoning.

Lesson 13 Summary

When working with exponents, the bases don't have to always be whole numbers. They can also be other kinds of numbers, like fractions, decimals, and even variables. For example, we can use exponents in each of the following ways:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$(1.7)^3 = (1.7) \cdot (1.7) \cdot (1.7)$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

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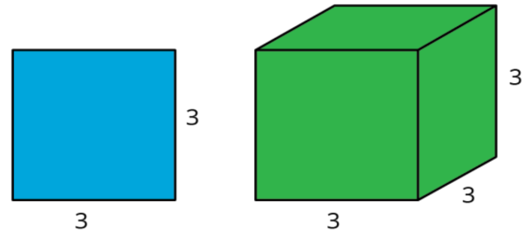
Unit 6, Lesson 14: Evaluating Expressions with Exponents

Let's find the values of expressions with exponents.

14.1: Revisiting the Cube

Based on the given information, what other measurements of the square and cube could we find?



14.2: Calculating Surface Area

A cube has side length 10 inches. Jada says the surface area of the cube is 600 in^2, and Noah says the surface area of the cube is 3,600 in^2. Here is how each of them reasoned:

Jada's Method:

$6 \cdot 10^2$
$6 \cdot 100$
600

Noah's Method:

$6 \cdot 10^2$
60^2
$3{,}600$

Do you agree with either of them? Explain your reasoning.

14.3: Expression Explosion

Evaluate the expressions in one of the columns. Your partner will work on the other column. Check with your partner after you finish each row. Your answers in each row should be the same. If your answers aren't the same, work together to find the error.

$5^2 + 4$	$2^2 + 25$
$2^4 \cdot 5$	$2^3 \cdot 10$
$3 \cdot 4^2$	$12 \cdot 2^2$
$20 + 2^3$	$1 + 3^3$
$9 \cdot 2^1$	$3 \cdot 6^1$
$\frac{1}{9} \cdot \left(\frac{1}{2}\right)^3$	$\frac{1}{8} \cdot \left(\frac{1}{3}\right)^2$

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Are you ready for more?

1. Consider this equation: $\square^2 + \square^2 = \square^2$. An example of 3 different whole numbers that could go in the boxes are 3, 4, and 5, since $3^2 + 4^2 = 5^2$ (That is, $9 + 16 = 25$). Can you find a different set of 3 different whole numbers that make the equation true?

2. How many sets of 3 different whole numbers can you find?

3. Can you find a set of 3 different whole numbers that make this equation true? $\square^3 + \square^3 = \square^3$

4. How about this one? $\square^4 + \square^4 = \square^4$

5. Once you have worked on this a little while, you can understand a problem that is famous in the history of math. (Alas, this space is too small to contain it.) If you are interested, consider doing some further research on *Fermat's Last Theorem*.

Lesson 14 Summary

Exponents give us a new way to describe operations with numbers, so we need to understand how exponents get along with the other operations we know.

When we write $6 \cdot 4^2$, we want to make sure everyone agrees about how to evaluate this. Otherwise some people might multiply first and others compute the exponent first, and different people would get different values for the same expression!

Earlier we saw situations in which $6 \cdot 4^2$ represented the surface area of a cube with side lengths 4 units. When computing the surface area, we evaluate 4^2 first (or find the area of one face of the cube first) and then multiply the result by 6. In many other expressions that use exponents, the part with an exponent is intended to be evaluated first.

To make everyone agree about the value of expressions like $6 \cdot 4^2$, the convention is to *evaluate the part of the expression with the exponent first*. Here are a couple of examples:

$6 \cdot 4^2$ $= 6 \cdot 16$ $= 96$	$45 + 5^2$ $= 45 + 25$ $= 70$

If we want to communicate that 6 and 4 should be multiplied first and then squared, then we can use parentheses to group parts together:

$$\begin{aligned}&(6 \cdot 4)^2\\ &= 24^2\\ &= 576\end{aligned}$$

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Unit 6, Lesson 15: Equivalent Exponential Expressions

Let's investigate expressions with variables and exponents.

15.1: Up or Down?

1. Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for different values of x.

x	3^x	$\left(\frac{1}{3}\right)^x$
1		
2		
3		
4		

2. What patterns do you notice?

15.2: What's the Value?

Evaluate each expression for the given value of x.

1. $3x^2$ when x is 10
2. $3x^2$ when x is $\frac{1}{9}$
3. $\frac{x^3}{4}$ when x is 4
4. $\frac{x^3}{4}$ when x is $\frac{1}{2}$
5. $9 + x^7$ when x is 1
6. $9 + x^7$ when x is $\frac{1}{2}$

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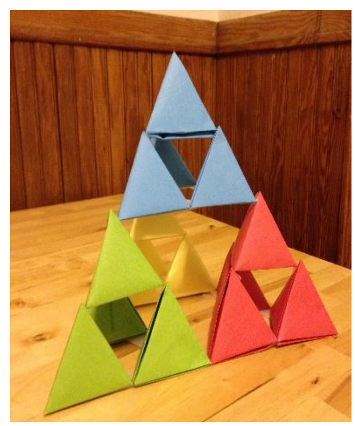
15.3: Exponent Experimentation

Find a solution to each equation in the list that follows. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

1. $64 = x^2$
2. $64 = x^3$
3. $2^x = 32$
4. $x = \left(\frac{2}{5}\right)^3$
5. $\frac{16}{9} = x^2$
6. $2 \cdot 2^5 = 2^x$
7. $2x = 2^4$
8. $4^3 = 8^x$

List: $\frac{8}{125}$ $\frac{6}{15}$ $\frac{5}{8}$ $\frac{8}{9}$ 1 $\frac{4}{3}$ 2 3 4 5 6 8

Are you ready for more?



This fractal is called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has four faces. (The plural of tetrahedron is tetrahedra.) The small tetrahedra form four medium-sized tetrahedra: blue, red, yellow, and green. The medium-sized tetrahedra form one large tetrahedron.

1. How many small faces does this fractal have? Be sure to include faces you can't see as well as those you can. Try to find a way to figure this out so that you don't have to count every face.

2. How many small tetrahedra are in the bottom layer, touching the table?

3. To make an even bigger version of this fractal, you could take four fractals like the one pictured and put them together. Explain where you would attach the fractals to make a bigger tetrahedron.

4. How many small faces would this bigger fractal have? How many small tetrahedra would be in the bottom layer?

5. What other patterns can you find?

Lesson 15 Summary

In this lesson, we saw expressions that used the letter x as a variable. We evaluated these expressions for different values of x.

- To evaluate the expression $2x^3$ when x is 5, we replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or just 250. So the value of $2x^3$ is 250 when x is 5.

- To evaluate $\frac{x^2}{8}$ when x is 4, we replace the letter x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So $\frac{x^2}{8}$ has a value of 2 when x is 4.

We also saw equations with the variable x and had to decide what value of x would make the equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: 1,2,3,9,11. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So 2 is the solution to the equation.

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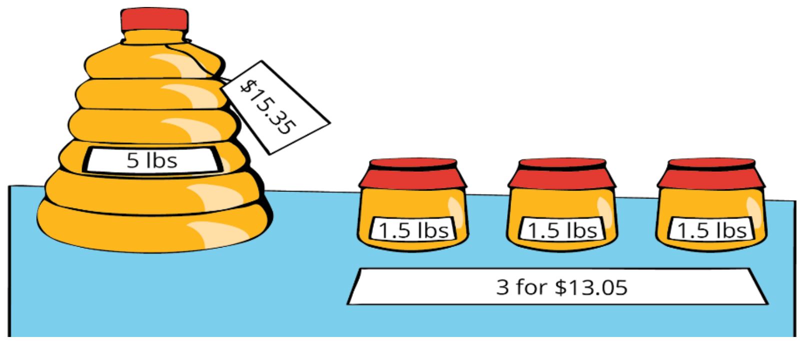
Unit 6, Lesson 16: Two Related Quantities, Part 1

Let's use equations and graphs to describe relationships with ratios.



16.1: Which One Would You Choose?

Which one would you choose? Be prepared to explain your reasoning.

- A 5-pound jug of honey for $15.35
- Three 1.5-pound jars of honey for $13.05

16.2: Painting the Set

Lin needs to mix a specific shade of orange paint for the set of the school play. The color uses 3 parts yellow for every 2 parts red.

1. Complete the table to show different combinations of red and yellow paint that will make the shade of orange Lin needs.

cups of red paint (r)	cups of yellow paint (y)	total cups of paint (t)
2	3	
6		
		20
	18	
14		
16		
		50
	42	

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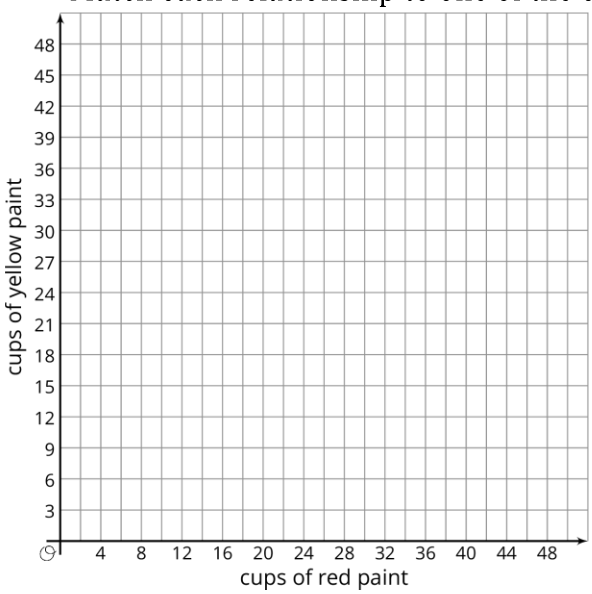
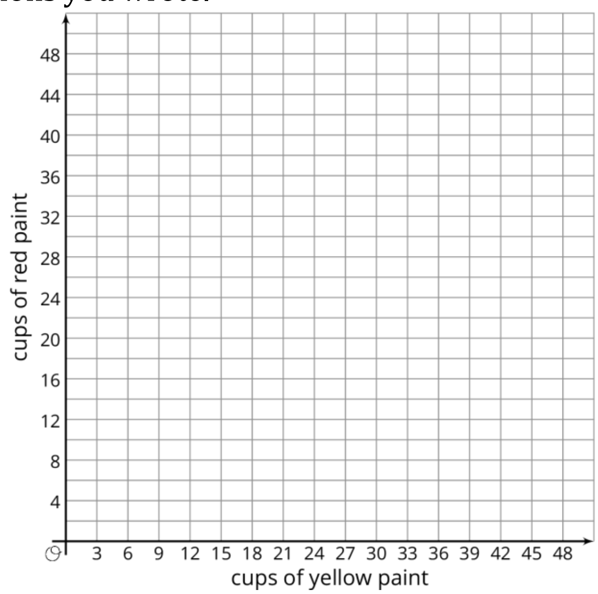
2. Lin notices that the number of cups of red paint is always $\frac{2}{5}$ of the total number of cups. She writes the equation $r = \frac{2}{5}t$ to describe the relationship. Which is the **independent variable**? Which is the **dependent variable**? Explain how you know.

3. Write an equation that describes the relationship between r and y where y is the independent variable.

4. Write an equation that describes the relationship between y and r where r is the independent variable.

5. Use the points in the table to create two graphs that show the relationship between r and y. Match each relationship to one of the equations you wrote.





Are you ready for more?

A fruit stand sells apples, peaches, and tomatoes. Today, they sold 4 apples for every 5 peaches. They sold 2 peaches for every 3 tomatoes. They sold 132 pieces of fruit in total. How many of each fruit did they sell?

Lesson 16 Summary

Equations are very useful for describing sets of equivalent ratios. Here is an example.

A pie recipe calls for 3 green apples for every 5 red apples. We can create a table to show some equivalent ratios.

green apples (g)	red apples (r)
3	5
6	10
9	15
12	20

We can see from the table that r is always $\frac{5}{3}$ as large as g and that g is always $\frac{3}{5}$ as large as r. We can write equations to describe the relationship between g and r.

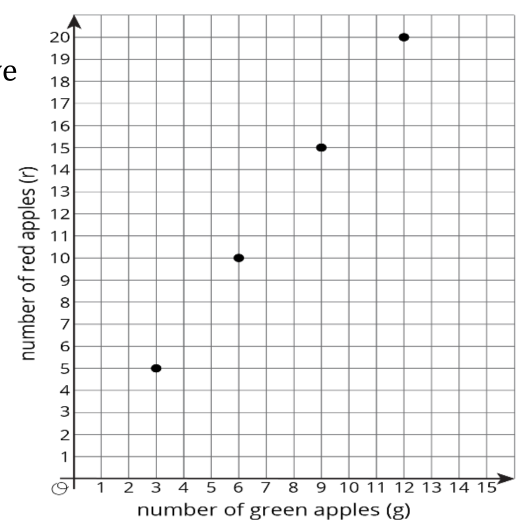
- When we know the number of green apples and want to find the number of red apples, we can write:
$$r = \frac{5}{3}g$$
In this equation, if g changes, r is affected by the change, so we refer to g as the **independent variable** and r as the **dependent variable**.

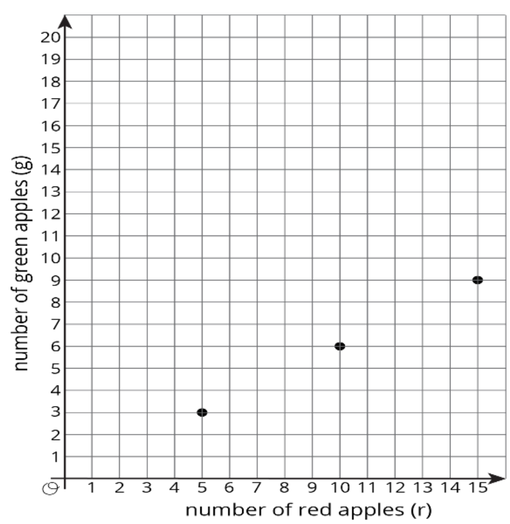
  We can use this equation with any value of g to find r. If 270 green apples are used, then $\frac{5}{3} \cdot (270)$ or 450 red apples are used.

- When we know the number of red apples and want to find the number of green apples, we can write:
$$g = \frac{3}{5}r$$
In this equation, if r changes, g is affected by the change, so we refer to r as the independent variable and g as the dependent variable.

  We can use this equation with any value of r to find g. If 275 red apples are used, then $\frac{3}{5} \cdot (275)$ or 165 green apples are used.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities.





Lesson 16 Glossary Terms

dependent variable independent variable

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Unit 6, Lesson 17: Two Related Quantities, Part 2

Let's use equations and graphs to describe stories with constant speed.

17.1: Walking to the Library

Lin and Jada each walk at a steady rate from school to the library. Lin can walk 13 miles in 5 hours, and Jada can walk 25 miles in 10 hours. They each leave school at 3:00 and walk $3\frac{1}{4}$ miles to the library. What time do they each arrive?

17.2: The Walk-a-thon

Diego, Elena, and Andre participated in a walk-a-thon to raise money for cancer research. They each walked at a constant rate, but their rates were different.

1. Complete the table to show how far each participant walked during the walk-a-thon.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1			
2	6		
	12	11	
5			17.5

2. How fast was each participant walking in miles per hour?

3. How long did it take each participant to walk one mile?

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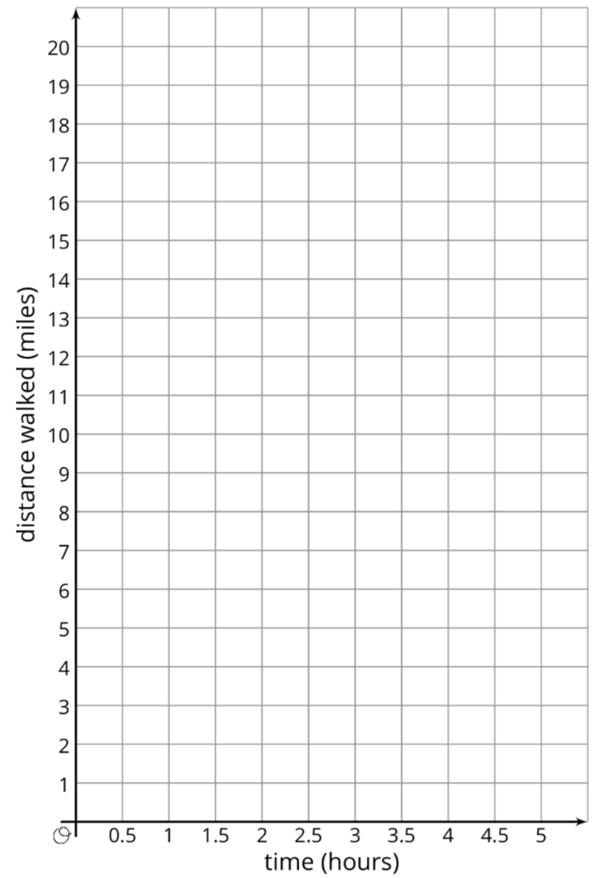
4. Graph the progress of each person in the coordinate plane. Use a different color for each participant.



5. Diego says that $d = 3t$ represents his walk, where d is the distance walked in miles and t is the time in hours.
    1. Explain why $d = 3t$ relates the distance Diego walked to the time it took.
    2. Write two equations that relate distance and time: one for Elena and one for Andre.

6. Use the equations you wrote to predict how far each participant would walk, at their same rate, in 8 hours.

7. For Diego's equation and the equations you wrote, which is the dependent variable and which is the independent variable?

Are you ready for more?

1. Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet?

    One way to start thinking about this problem is to make a table. Add as many rows as you like.

	Train A	Train B
Starting position	0 miles	320 miles
After 1 hour	70 miles	270 miles
After 2 hours		

2. How long will it take a train traveling at 120 miles per hour to go 320 miles?

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3. Explain the connection between these two problems.

Lesson 17 Summary

Equations are very useful for solving problems with constant speeds. Here is an example.

A boat is traveling at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?
2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these. Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

1. When we know the time and want to find the distance, we can write:
$$d = 25t$$
In this equation, if t changes, d is affected by the change, so we t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t. In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.

2. When we know the distance and want to find the time, we can write:
$$t = \frac{d}{25}$$
In this equation, if d changes, t is affected by the change, so we d is the independent variable and t is the dependent variable.

This equation can help us find t when for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

These problems can also be solved using important ratio techniques such as a table of equivalent ratios. The equations are particularly valuable in this case because the answers are not round numbers or easy to quickly evaluate.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities: